Title: Charting a New Course

Link to Outcomes:

- **Connections** Students will show the relationship of geometry to geography and the applications of technology.
- **Communication** Students will express their ideas, calculations, and results in a clearly written grade-appropriate manner.
- **Reasoning** Students will be able to experiment and establish conclusions based on the collected data, calculations and theorems.
- **Measurement** Students will use the TI Calculator Based Laboratory (CBL) system, a ruler, and calculation to determine distances.
- **Statistics** Students will organize data on a table and as data plots on a graph.
- **Technology** Students may use the TI-82 graphics calculator to graph data and to perform calculations. Students may use the CBL-Ultrasonic Motion Detector to measure lengths needed in the experimentation.
- **Real-World Applications** Students will use their knowledge of Geometric concepts to plan a vacation, to travel certain distances, or to calculate area in order to seed a plot of land.

Brief Overview:

Students learn to use the CBL and the Ultrasonic Motion Detector and they learn to become proficient with the TI-82. Students gather data from the Ultrasonic Motion Detector to determine lengths of sides of triangles. Using the Motion Detector and calculation, students determine the hypotenuse with that information and can calculate areas of the triangles. Students are expected to verify the length of the hypotenuse by using the Pythagorean Theorem. These results are used to find the hypotenuse of a much larger similar triangle.

Grade/Level:

Grades 7-12, Geometry/ Algebra/ Algebra II (Extension only)

Duration/Length:

This activity will take 2 class periods to complete.

Prerequisite Knowledge:

- Ability to calculate area of triangles
- Ability to solve proportions
- Ability to measure accurately
- Working knowledge of the TI-82 graphics calculator
- Ability to use the Pythagorean Theorem

Objectives:

Students will be able to:

- Apply the Pythagorean Theorem to solve for unknown sides in a triangle.
- Calculate length, perimeter, or area knowing either length, perimeter, or area of a similar figure, given information on the scale factor of the two figures.
- Apply properties of similar triangles.
- Apply the relationship between the length of the sides and the area of similar right triangles.

Materials/Resources/Printed Materials:

- Masking tape, ruler, scissors, string
- Student worksheets
- TI-82 graphics calculator
- CBL/Ultrasonic Motion Detector

Development/Procedures:

On day 1, students observe a demonstration of the CBL and Ultrasonic motion detector. Using information from day one, they make measurements and collect data to complete **Worksheet #1**. These measurements become the legs of triangles which the students use to determine the hypotenuse by two methods: direct measurement and Pythagorean Theorem. Students compute the areas of the triangles and add these values to **Worksheet #1**. After completing the worksheet, students answer questions based on the accuracy of their data and calculations.

On day 2, students work on activity sheets to assess their knowledge of right triangles, of the Pythagorean Theorem, and of similar triangles. Students help fictional characters plan a vacation route to minimize travel time and distance, find distances based on the properties of similar triangles, and seed a plot of land.

As an extension to the lesson, students calculate area and measure the hypotenuse of several similar triangles, and graph area as a function of the hypotenuse using the TI-82 calculator.

Evaluation:

The assessment for this lesson may be used either formatively or summatively. When used as formative, the teacher distributes all of the worksheets as in-class group activity sheets. When used as summative, the teacher distributes some of the worksheets for in-class activities and others for assessment purposes. Good performance assessment worksheets are **Worksheet #2** and **Worksheet #3**.

Extension/Follow Up:

- **Worksheet #5** is an extension to the lesson. It could be used during a third day, if there is time or it could be given to more highly motivated students.
- The lesson could be expanded to include other polygons or three dimensional figures.

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TEACHER'S GUIDE

OVERVIEW:

Students learn to use the CBL and the Ultrasonic Motion Detector and to become proficient with the TI-82. Students gather data from the Ultrasonic Motion Detector to determine lengths of sides of triangles. Using the Motion Detector and calculation, students determine the hypotenuse and areas of the triangles. Students are expected to verify the length of the hypotenuse by using the Pythagorean Theorem. These results are used to find the hypotenuse of a much larger triangle. Students compare the sizes of the similar triangles.

MATERIALS:

- Masking tape, ruler, scissors, string
- Student worksheets
- TI-82 graphics calculator
- CBL/Ultrasonic Motion Detector

PROCEDURE:

Activity 1:

1. Hand out **Worksheet #1** and allow students time to read the activity.
2. Put students in groups of 2–4, depending on how many CBL's are available.
3. Distribute one CBL and some masking tape to each group of students.
4. Students connect the Ultrasonic Motion Detector to the CBL by inserting the connector from the Motion Detector into the **SONIC** port on the CBL.
5. Since measurements will be recorded in feet, press hard on the **CHANNEL VIEW** button on the CBL. Scroll through the displayed symbols until **{soNic}Ft** is displayed in the middle of the screen.
6. Students point the motion detector at a wall until a length of 3 feet is displayed on the CBL. Students cut a piece of masking tape whose length spans from the wall to the CBL.
7. Step #4 is repeated for 4 feet.
8. Picking a space on a wall, students place one length of masking tape on the wall so that one end just touches and is perpendicular to the floor.
9. Students place the other length of masking tape on the floor so that one end just touches and is perpendicular to the first piece of masking tape.
10. Students cut a piece of string so that it will be long enough to connect the endpoints of the masking tape, to form the hypotenuse of a right triangle.
11. Students make four additional measurements with the CBL and complete **Worksheet #1** by filling in the data table and answering the questions.

Activity 2:

1. Hand out **Worksheet #2** and allow students time to read the scenario.
2. Students complete **Worksheet #2** (which tests a student's knowledge of right triangles and the Pythagorean Theorem in a real life situation).
3. If this is used as a formative activity, the students may work in groups.
4. Students should be shown the rubric for the best paper, so they understand the criteria on which they will be scored. Students complete the activity individually.

Rubrics for **Worksheet #2:**

3: This student calculates hypotenuse accurately. The student presents correct derivation of the answer. The diagram is neat and clear. The letter is written convincingly with correct use of the language.

2: This student calculates hypotenuse accurately. The student may present correct derivation of the answer. The diagram is reasonably neat and clear. The letter is written clearly with correct use of the language.

1: This student calculates hypotenuse accurately. The student presents some derivation of the answer. The diagram is moderately neat. The letter is written somewhat clearly with fair use of the language.

0: This student does not calculate hypotenuse correctly. The student presents very little derivation of the answers. The diagrams are not clear and the student does not use references to Worksheet #1. The letter is not written clearly or convincingly.

Activity 3:

1. Hand out **Worksheet #3** and allow students time to read the scenario.
2. Review the concept of similarity of right triangles.
3. Students complete **Worksheet #3** which assesses their knowledge of ratio, proportion and similarity using right triangles.
4. If this is used as a formative activity, the students may work in groups.
5. Students should be shown the rubric for the best paper, so they understand the criteria on which they will be scored. Students complete the activity individually.

Rubrics for **Worksheet #3:**

3: The student accurately and correctly draws the original right triangle with legs labelled. The student accurately shows the new route to Kentucky with the legs 2/3 of the larger lengths. The miles calculated are the sum of the legs of the new triangle.

2: The student draws the original triangle with most measures shown. The student shows the new route to Kentucky with the legs 2/3 of the larger length. The miles calculated are approximately the sum of the legs of the new triangle.

1: The student draws one of the triangle's inaccurately. The smaller triangle has legs 2/3 of the original triangle but the triangles are not labelled. The mileage to Kentucky is not the sum of the new legs.

0: The student draws an inaccurate triangle. The legs are not labelled. The route to Kentucky is not 2/3 of the original legs. No miles are calculated.

Activity 4:

1. Hand out **Worksheet #4** and allow students time to read the scenario.
2. If necessary, review the concepts of perimeter and area.
3. Students complete **Worksheet #4** which assesses their ability to calculate perimeter and area of right triangles.
4. The activity uses the real life activity of calculating the amount of grass seed to purchase for the area of the property.
5. If this is used as a formative activity, the students may work in groups.
6. Students should be shown the rubric for the best paper, so they understand the criteria on which they will be scored. Students complete the activity individually.

Rubrics for **Worksheet #4:**

3: This student calculates the perimeter and area of the shaded region and the costs of the grass seed. Work is consistently shown clearly and correctly. The student gives a convincing explanation of similarity and clearly explains the relationship between lines and areas of similar figures.

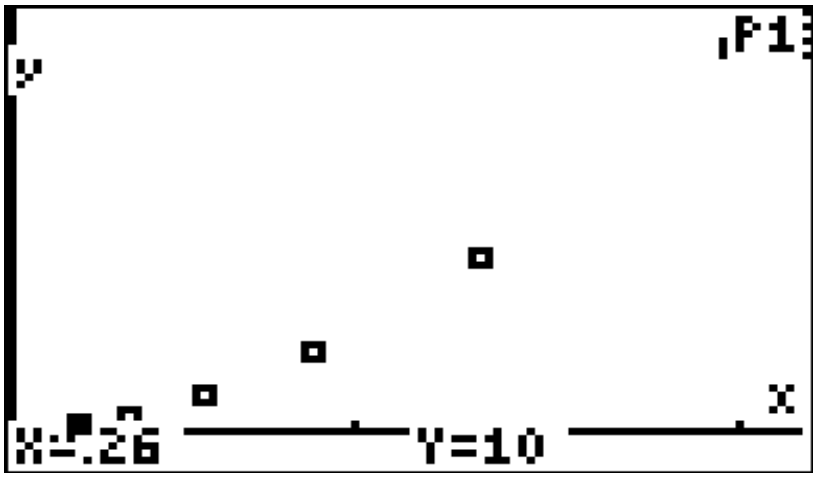
2: This student frequently calculates the perimeter and area of the shaded region and the costs of the grass seed. Work is often shown correctly. This student gives a somewhat convincing explanation of similarity and explains the relationship between lines and areas of similar figures.

1: This student sometimes calculates the perimeter and area of the shaded region and the costs of the grass seed. Work is sometimes shown correctly. This student gives a weak explanation of similarity and somewhat explains the relationship between lines and areas of similar figures.

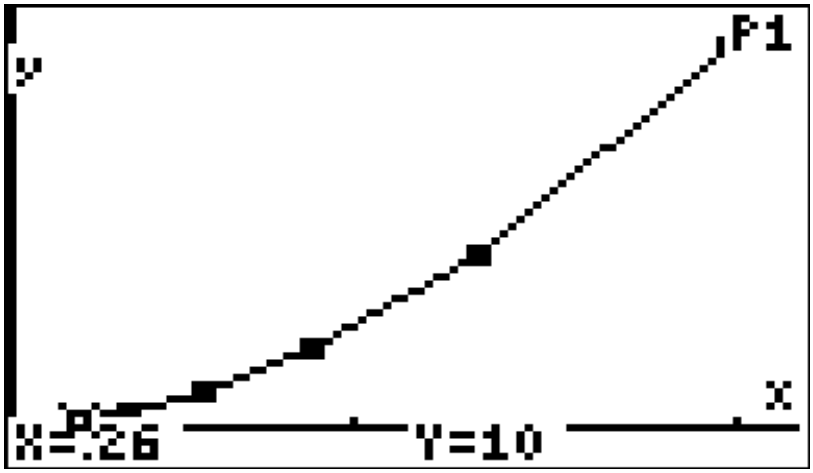
0: This student rarely calculates the perimeter and area of the shaded region and the costs of the grass seed. Work is occasionally shown. This student gives a poor explanation of similarity and rarely explains the relationship between lines and areas of similar figures.

Activity 5:

1. **Worksheet #5** is an extension of the activities contained in this learning unit.
2. Hand out **Worksheet #5** and allow students time to read the scenario.
3. If necessary, you may review the concepts of linear and quadratic graphing.
3. Students make calculations based on the information given on the worksheet and complete the data table and answer questions.
4. **Worksheet #5** assesses knowledge of ratio, proportion and similarity using right triangles. It also assesses ability to use the TI-82 to input data and to graph linear and quadratic functions.
5. The data plot on the TI-82 should match the following graphs:



PLOTTED DATA POINTS



DATA POINTS CONNECTED

WORKSHEET #1

Directions: Use the CBL-Ultrasonic Motion Detector to determine the lengths of the legs of each of your triangles, including those given. Mark these legs using masking tape on the wall and floor. For the missing legs, use the motion detector to create your own numbers. Determine the lengths of each hypotenuse with string. Fill in all missing data on the table below. Finally, calculate the area of each of your triangles.

Trial	Leg 1	Leg 2	Hypotenuse	Area
1	3 ft.	4 ft.		
2	2 ft.	3 ft.		
3	4 ft.			
4	1.5 ft			
5				

Questions:

1. Verify each entry in the hypotenuse column by using the Pythagorean Theorem. Show your work in the space below.

2. How close were your measurements in the table compared to your results in question #1? Explain your results.

WORKSHEET #2

Your friend, Wild Bill Snow, a Texas rancher, is planning a trip to Washington, D.C. for spring vacation. He has sent you his planned route. When you see that he will be travelling 1200 miles due East, then 750 miles due North, you realize that he could save lots of time, gas, and mileage if he drove the hypotenuse of the triangle. The distance from coast to coast in the United States is approximately 3000 miles.

Write a letter to Wild Bill in which you try to convince him to drive the hypotenuse. Use diagrams to add to your argument.

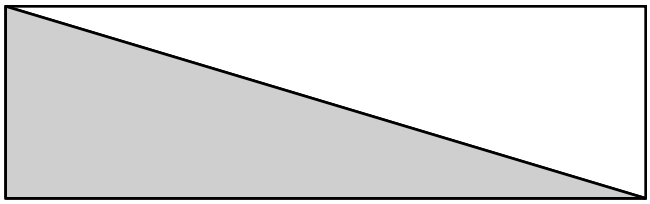
WORKSHEET #3

Wild Bill has a sister, Smooth Sally, a Country Music Singer who lives in Kentucky. She would like to accompany Wild Bill on his trip to Washington. She lives two-thirds of the way to Washington, along a line that connects Wild Bill's ranch and the Capital. If Wild Bill had kept to his original route, how many miles would he have travelled to pick up his sister? Remember, his original route was due East then due North.

Draw a map of Wild Bill's original route and the new route to Kentucky. Illustrate the measure of each leg.

WORKSHEET #4

While Wild Bill was in Washington, D.C. and you were talking to him, he invited you to work on his ranch over the summer. Bill has just purchased new property adjacent to his ranch. His intent is to plant grass so that his cattle will be able to graze on half of the new rectangular piece of land. He will need help in fencing the land, in calculating the area, and in figuring out how much grass seed he must purchase. The diagram below shows the amount of land that he is planning to seed.



You want to do some planning before you go to Texas next summer, so you find out the following:

1. The length of the above rectangle is 3000 feet.
2. The width of the rectangle is 2000 feet.
3. Grass seed costs $1.25 per square foot.

Calculate the perimeter and area of the shaded area. Show all of your work. Determine the costs that Wild Bill will incur in grass seed and show your work.

Go back to **Worksheet #1** and compare your answers above to the answers in line 2. Compare and contrast the size of the legs, the size of the hypotenuse, the areas on the two worksheets.

WORKSHEET #5

Wild Bill noticed while he was travelling to Washington that Maryland is a rather small state that has a shape that is roughly a right triangle. Its border with Pennsylvania and Delaware are the legs and the Photonic River is the hypotenuse. Since Wild Bill is from Texas, he was curious to find out how much he would have to multiply the borders of Maryland by to give it the same area as Texas.

Figure out the measure of the hypotenuse for line 1. Fill out the chart below by multiplying the given values on line 1 by a factor of 1.5. On each successive line repeat the process by multiplying the values of the legs and hypotenuse on the previous line by 1.5. Figure the area of each triangle as you go along. (round answers to 2 decimal places)

After your table is finished, plot the area as a function of the hypotenuse values with your TI-82. (Graph hypotenuse divided by 1000 and area divided by 1000 for this application.)

Line #	Short Leg	Long Leg	Hypotenuse	Area
1	**100 mi.**	**400 mi.**		
2				
3				
4				
5				
6				
7				

1. How much did Wild Bill have to multiply the sides by to make Maryland approximately the same size as Texas? Texas is 261,000 square miles big.

2. What sort of graph did you get when you graphed area as a function of the hypotenuse?

Appendix
Rubrics

Rubrics for **Worksheet #2:**

- **3:** This student calculates hypotenuse accurately. The student presents correct derivation of the answer. The diagram is neat and clear. The letter is written convincingly with correct use of the language.
- **2:** This student calculates hypotenuse accurately. The student may present correct derivation of the answer. The diagram is reasonably neat and clear. The letter is written clearly with correct use of the language.
- **1:** This student calculates hypotenuse accurately. The student presents some derivation of the answer. The diagram is moderately neat. The letter is written somewhat clearly with fair use of the language.
- **0:** This student does not calculate hypotenuse correctly. The student presents very little derivation of the answers. The diagrams are not clear and the student does not use references to Worksheet #1. The letter is not written clearly or convincingly.

Rubrics for **Worksheet #3:**

- **3:** The student accurately and correctly draws the original right triangle with legs labelled. The student accurately shows the new route to Kentucky with the legs 2/5 of the larger lengths. The miles calculated are the sum of the legs of the new triangle.
- **2:** The student draws the original triangle with most measures shown. The student shows the new route to Kentucky with the legs 2/5 of the larger length. The miles calculated are approximately the sum of the legs of the new triangle.
- **1:** The student draws one of the triangle's inaccurately. The smaller triangle has legs 2/5 of the original triangle but the triangles are not labelled. The mileages to Kentucky is not the sum of the new legs.
- **0:** The student draws an inaccurate triangle. The legs are not labelled. The route to Kentucky is not 2/5 of the original legs. No miles are calculated.

Rubrics for **Worksheet #4:**

- **3:** This student calculates the perimeter and area of the shaded region and the costs of the grass seed. Work is consistently shown clearly and correctly. The student gives a convincing explanation of similarity and clearly explains the relationship between lines and areas of similar figures.
- **2:** This student frequently calculates the perimeter and area of the shaded region and the costs of the grass seed. Work is often shown correctly. This student gives a somewhat convincing explanation of similarity and explains the relationship between lines and areas of similar figures.
- **1:** This student sometimes calculates the perimeter and area of the shaded region and the costs of the grass seed. Work is sometimes shown correctly. This student gives a weak explanation of similarity and somewhat explains the relationship between lines and areas of similar figures.
- **0:** This student rarely calculates the perimeter and area of the shaded region and the costs of the grass seed. Work is occasionally shown. This student gives a poor explanation of similarity and rarely explains the relationship between lines and areas of similar figures.